EDGES

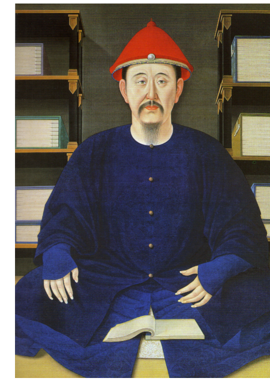
In Which the Mathematics of Japanese Stab Bookbinding is Investigated

Natalie Freed, 2015

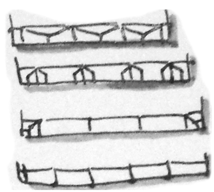
The “stab” or “stitched binding” technique (*hsien chuang*) originated in China during the Ming Dynasty (1573-1619) as an evolution of thousands of years of book development: from bamboo and wood surfaces, to silk and paper scrolls, to glued bindings, and finally to an elegant style that did not require glue and was easy to repair.

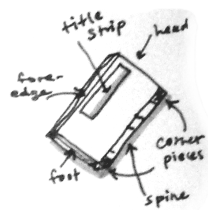
The technique spread to Japan along with other key book technologies such as paper, ink, and woodblock printing techniques. The Japanese developed several variations on the style.

Traditional Japanese designs include the 4-hole, Kangxi, tortoiseshell, and hemp leaf patterns (bottom to top). Chinese patterns tend to have more asymmetry (in spacing between holes) and are constructed with more flexible covers.



The Chinese emperor Kangxi was said to have invented a binding style with reinforcing corners. Notice the open book and the protective boxes.

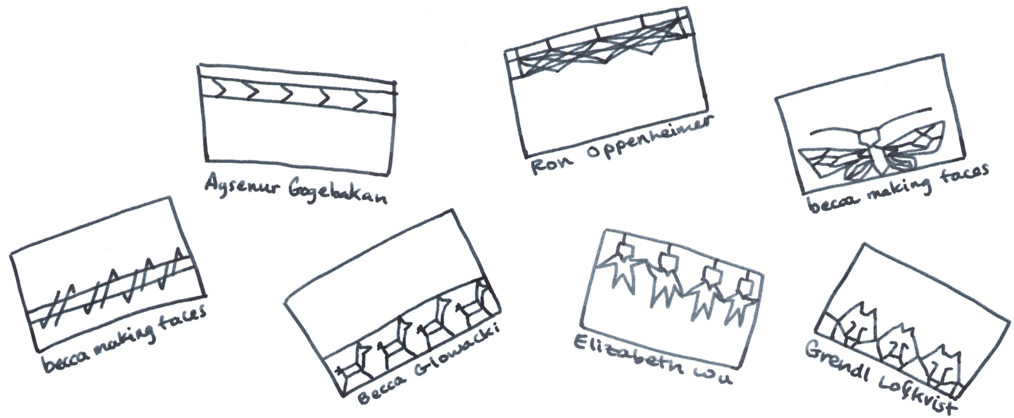




This binding technique is especially well-suited to flexible and strong Chinese & Japanese paper, made from mulberry and other plants.

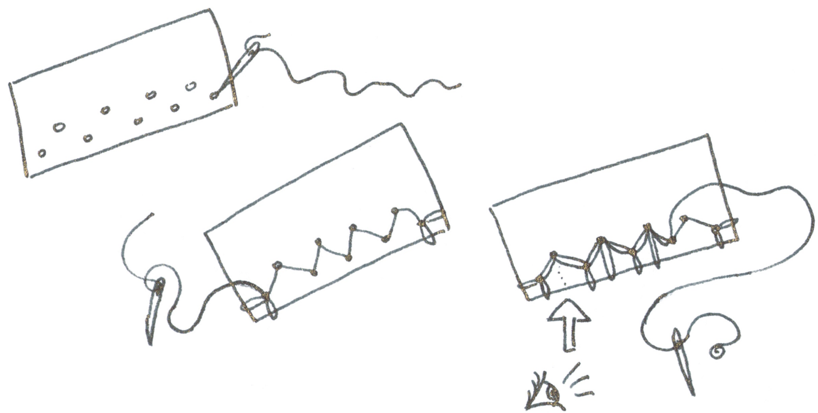
Contemporary Stab Bindings

Modern examples branch out of the traditional styles to explore figurative designs and fancy geometric patterns, sometimes emphasizing beauty over practicality!



These particular books were done in the western rather than original Asian style, bound on the left for reading left to right (though only the spines are pictured here).

The fancier the patterns get, the trickier it is to stitch them! At each step you have to decide which path to take through the pattern. If you skip a stitch and go too far, you can't go back without doubling up a stitch.

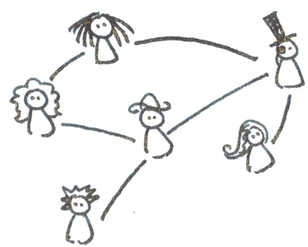


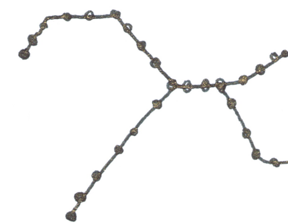
Tutorials online for stab patterns have pictures and helpful step by step directions ("start at hole 3, go to hole 5"...), but how do you invent your own? How do you know which designs will actually work with a single piece of thread? Is there a more succinct way to write instructions?

The answer lies in the realm of "graph theory." A **graph** is a mathematical representation of a set of things that can have connections between them. Not to be confused with XY plots… mathematicians regret naming these with the same term!

vertex (plural: vertices) -> a "thing" or object in the graph (dot).
edge -> a link or connection between two vertices (line).

vertices could represent people and edges the relationships between them.



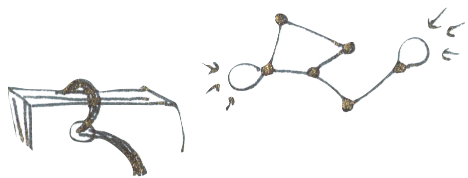


edges could symbolize train routes and vertices the stations.

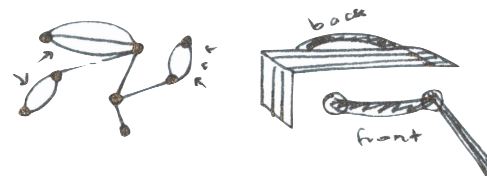
vertices could be holes and edges the stitches connecting them!

Some special features we will need for stab binding graphs:

loops: edges that connect a vertex to itself. In the books, these are stitches that wrap around the edge and back into the same hole.



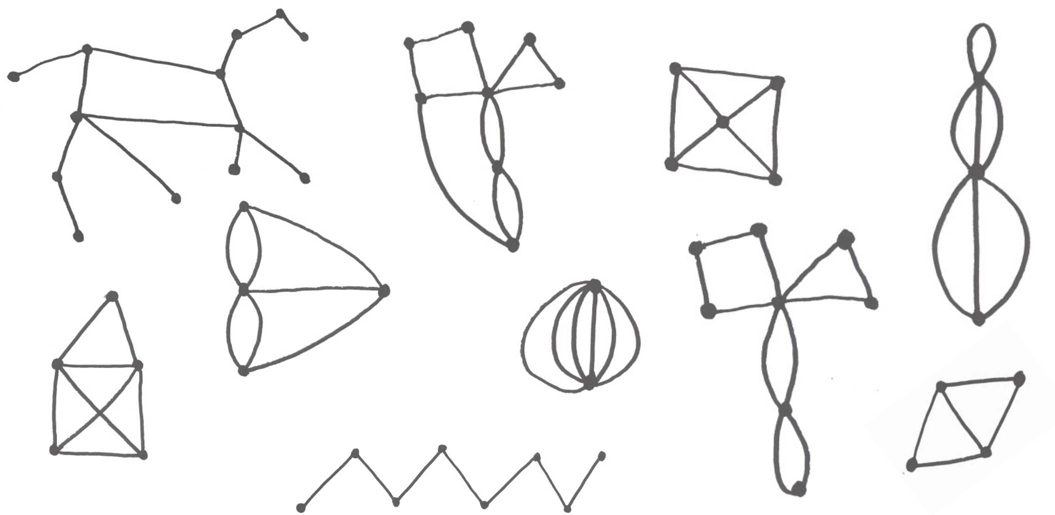
parallel edges: multiple edges that connect the same two vertices to each other. We will need these in order to have a front and back stitch between each set of holes.



Graphs that allow these two properties are called pseudographs or multigraphs (not all mathematicians use the same terminology...)

Now that we can draw a stitch pattern as a graph, we need to figure out how to travel through that graph from one vertex (hole) to another without skipping any edges (stitches).

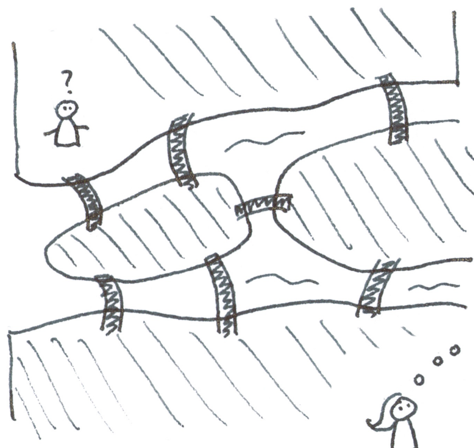
Graph puzzle! Starting at one vertex and following edges without lifting your pen, can you trace all the edges of each graph? (drawings by B.R.G.)



Are some of them impossible? How can you know for SURE?

This turns out to be the question that founded the field of graph theory.

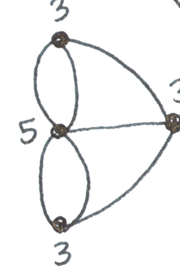
There was once a city called Königsberg in old Prussia that had two islands in a river between two parts of the city, all connected by seven bridges.
The famous question: Could you a plan a route through the city that crossed each bridge exactly once?



the city as a graph ->

A mathematician named Leonhard Euler solved it, and here was his answer:

Nope, impossible! Here's how you can know for sure:
Count all the edges connected to each vertex, and
call that number the "degree" of the vertex.

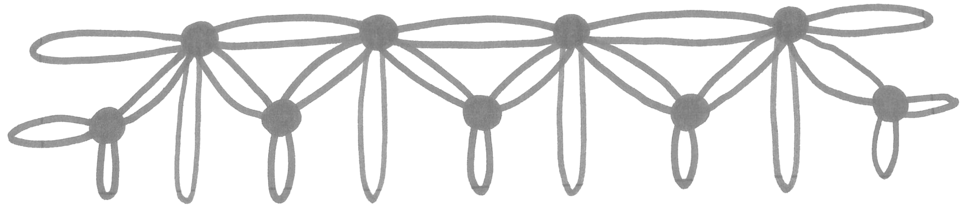


If the degree of all vertices is even, there is a route
through all the edges that returns to its starting point.
That's an Euler circuit.

If the degree of exactly two vertices is odd, there is a route through
all the edges that starts at one of the odd vertices and ends at the
other. That's an Euler trail.

If there is any other number of odd-degree vertices:
It can't be done!

What does this tell us about our books?



Since in a traditional stab book every edge is doubled (for the front and back stitches), every hole will always have an even number of connections reaching it. So there will always be an Euler circuit, and thus always a way to stitch it.

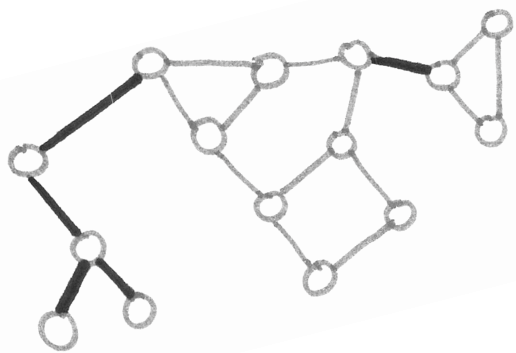
Awesome!! Assuming we stick to the same pattern on both sides of our stab books, we know that *any design we come up with will be possible to stitch!*

But! How do we know the stitch order?

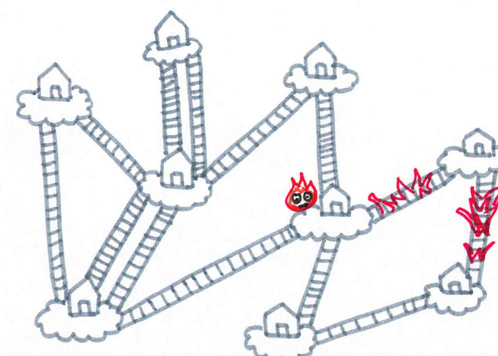
There are a number of different algorithms. The one that follows is neat because it helps you decide where to stitch one step at a time and gives you choices along the way.

First, another puzzle:

what is special about the edges marked in bold?



Which ladder should the fire demon take next if he wants to reach every ladder to stop a spreading plague?



Those bolded edges are called “bridges” or “cut edges,” and their significance is that if you remove them, the graph will be broken into two pieces with no way to get from one to another.

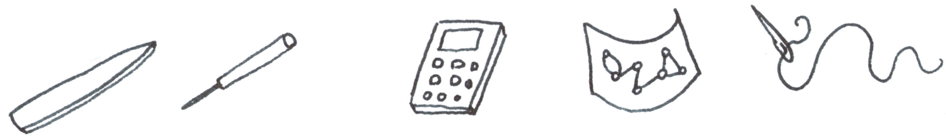
Fleury’s algorithm for finding Euler circuits in a graph goes like this:

Don’t burn bridges.

That’s it? In more detail: as you make your way through the graph, only take an edge that is a bridge if it is your only option. Otherwise, take any other edge. This algorithm will find a Euler circuit in any graph that has one (and gives you route options along the way).

How this relates to stab bindings: before you make a stitch, check if that stitch is the only connection to some part of your pattern. If it is, take another route if you can. At some point, you will come back to that hole and the stitch you were going to take will be your only option. At which point, go ahead and burn the bridge!

Where to from here? Now you can design your own!



Here's an app that will help you calculate the stitch pattern using the algorithm we described: nataliefreed.github.io/stab-bindings-designer

But technology is only one tool in an artist's hand.

Trickier things include designing a pattern you're happy with and which holds your book together well, decisions about proportions and layout and color and content, choosing a harmonious combination of materials for book and thread, the craftsmanship and technique, the meaning and narrative.

A few guidelines specific to stab books: leave enough space between holes that they don't weaken the book, reinforce where the cover folds open (notice the line across in all the traditional patterns), and don't put too many stitches through one hole!

Sources & Resources

Graphs and Euler Paths:

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Stab Bindings: beccamakingfaces.com/category/japanese-stab-binding

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homepages.nildram.co.uk/~dawe5/bookbinding_pages/BB_welcome2.html

Chinese Traditional Bookbinding: A Study of Its Evolution and Techniques by Edward Martinique

Credits

This exploration is by Natalie Freed and Becca Rose Glowacki. We were taking a bookbinding course together at San Francisco City College. Our wonderful teacher Grendl Lofkvist showed us examples of stab bindings, including a cat pattern she had stitched. We wondered if we could compute which stab patterns were possible to stitch, and started investigating.

Thank you to Forrest Green for programming help and to Paula Hooper, Meg Escudé, and the Exploratorium Tech crew for design feedback and working with us on curriculum development.